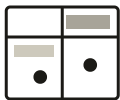


PROBLEM PACKET

Unit 2A

Probability



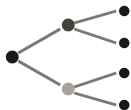
TWO-WAY TABLES



SIMULATION



VENN & SETS



TREE DIAGRAMS

READ THIS FIRST

How this packet works

This is a **problems-only** packet — your notes and worked examples live in the Guided Notes. Unit 2 is about **chance**: how to measure it, model it, and reason with it. Part 1 (this packet) builds the foundation — **reading two-way tables, running simulations, and applying the rules of probability**. Work runs in five short modules (A–E): organize categorical data in **two-way tables**, estimate probabilities by **simulation**, apply the **basic rules**, handle **conditional probability & independence**, and finish with **unions, the general addition rule, and tree diagrams**. Inside each module, problems climb from warm-up to exam-level and mix short answer, multiple choice, and free response. **Show the structure of every probability calculation** — an expression, the numbers substituted in, and an answer. Difficulty is tagged with stars:

- ★★★★

WARM-UP — recall & one step
- ★★★★★

PROFICIENT — standard multi-step skill
- ★★★★★

ADVANCED — true exam level: explain & justify
- ★★★★★

CHALLENGE — hardest exam reasoning

Two-Way Tables & Categorical Association

Two-way (contingency) tables · joint, marginal & conditional relative frequencies · side-by-side & segmented bar charts · judging association

1 True or False. For any you mark false, give a one-line reason.

★★★★

- a) A two-way table summarizes data for two categorical variables at once.
- b) A joint relative frequency is a single cell count divided by the table’s grand total.
- c) A marginal relative frequency is found by dividing a cell count by its row total only.

USE THIS TABLE FOR PROBLEMS 2–7

A school surveyed 200 students, asking whether they subscribe to a music-streaming app and whether they have a part-time job.

	JOB: YES	JOB: NO	TOTAL
Streaming: Yes	40	90	130
Streaming: No	40	30	70
TOTAL	80	120	200

2 Find each **marginal** relative frequency. (a) What proportion of the students subscribe to streaming? (b) What proportion have a part-time job?

★★★

3 (a) Find the **joint** relative frequency of students who stream *and* have a job. (b) Complete the joint relative-frequency table below.

★★★

	JOB: YES	JOB: NO	TOTAL
Streaming: Yes			
Streaming: No			
TOTAL			

- 4 Find each **conditional** relative frequency. (a) Among students *with* a job, what proportion stream? (b) Among students *without* a job, what proportion stream?

★★★

- 5 Using your answers to problem 4, is there an **association** between having a part-time job and subscribing to streaming? Justify your answer by comparing the two conditional relative frequencies.

★★★

- 6 For these 200 students, which calculation gives a **conditional** relative frequency?

★★★



- A) $40 / 80$
- B) $130 / 200$
- C) $80 / 200$
- D) $40 / 200$
- E) $200 / 200$

- 7 Using the two-way table for problems 2–7, give the **conditional distribution** of streaming status (Yes / No) for students *with* a job and for students *without* a job. Then state whether streaming status appears **associated** with having a job, and explain briefly.

★★★

USE THIS TABLE FOR PROBLEMS 8–9

A college dining hall asked 180 students which meal plan they use and which year they are.

	SOPHOMORE	JUNIOR	SENIOR	TOTAL
Unlimited plan	36	24	12	72
Block plan	24	30	18	72
No plan	12	6	18	36
TOTAL	72	60	48	180

- 8** (a) What proportion of all surveyed students are seniors? (b) Among seniors, what proportion have no meal plan?
 ★★★ (c) Among students with no meal plan, what proportion are seniors? (d) Explain why your answers to (b) and (c) are different even though both involve seniors and “no plan.”

- 9** Does the meal plan a student uses appear to be **associated** with their year? Support your answer by computing the conditional distribution of meal plan for at least two of the year groups and comparing them.
 ★★★

- 10** In a two-way table, a **marginal** relative frequency is

★★★★



- A) a single cell count divided by the grand total.
 B) a row total or column total divided by the grand total.
 C) a cell count divided by its row total.
 D) a cell count divided by its column total.
 E) always equal to 0.5.

- 11** **Error Analysis.** A student is asked, “Among students with a job, what fraction stream?” and writes “40 / 130, because 40 stream-and-work and 130 stream in total.” Using the table from problems 2–7, explain the student’s mistake and give the correct calculation and value.
 ★★★

12 **True or False.** For any you mark false, give a one-line reason.

★★★

- a) All of the joint relative frequencies in a two-way table add up to 1.
- b) If the conditional distributions of one variable are the same across every level of the other, the two variables are associated.
- c) A segmented bar chart and a mosaic plot can both display the relationship between two categorical variables.

13 A segmented bar chart shows the conditional distribution of “owns headphones” for commuters and non-commuters. The two segments for “Yes” are clearly different heights. This is evidence that

★★★★



- A) commuting causes students to own headphones.
- B) owning headphones and commuting are independent by definition.
- C) owning headphones and commuting are associated.
- D) the sample size was too small.
- E) the two variables are the same variable.

14 **Matching.** Using the table from problems 2–7, match each phrase to its calculation. Each is used once.

★★★

- | | |
|--|---------------|
| 1. Joint relative frequency (stream & job) | A. $40 / 80$ |
| 2. Marginal relative frequency (job) | B. $80 / 200$ |
| 3. Conditional rel. freq. (stream, given job) | C. $40 / 130$ |
| 4. Conditional rel. freq. (job, given streams) | D. $40 / 200$ |

15 A streaming service studied whether finishing a show’s first episode is associated with the device used. Of 300 viewers: of 180 phone viewers, 72 finished episode 1; of 120 TV viewers, 78 finished episode 1.

★★★★

- a) Organize the counts in a complete two-way table (device by finished / did not finish), including all totals.
- b) Find the conditional relative frequency of finishing episode 1 for each device.
- c) Is device associated with finishing the first episode? Justify using your part (b) values.

	FINISHED	DID NOT	TOTAL
Phone			
TV			
TOTAL			

- 16** Explain the difference between a **joint** relative frequency and a **conditional** relative frequency. Use a single cell of a two-way table to show how the two calculations differ (what changes in the denominator).

USE THIS TABLE FOR PROBLEMS 17–22, 24–25 & 28

A survey of 240 high-school students asked whether they prefer to text or to call a friend, recorded by class level.

	UNDERCLASSMAN	UPPERCLASSMAN	TOTAL
Prefers texting	90	60	150
Prefers calling	30	60	90
TOTAL	120	120	240

- 17** Find each value. (a) The **marginal** relative frequency of preferring texting. (b) The **joint** relative frequency of preferring texting *and* being an underclassman.

- 18** Find each **conditional** relative frequency. (a) Among underclassmen, what proportion prefer texting? (b) Among upperclassmen, what proportion prefer texting? Show each calculation.

- 19** Is class level **associated** with texting-vs-calling preference? Justify by comparing the conditional distributions from problem 18.

20 Which display best shows whether class level is associated with preference?

★★★



- A) A pie chart of all 240 students by preference.
- B) A dotplot of the 240 students.
- C) A single bar showing the total number who prefer texting.
- D) A segmented (100%) bar chart of preference for each class level.
- E) A histogram of class levels.

21 Using the table, which calculation is a **conditional** relative frequency?

★★★



- A) $150 / 240$
- B) $90 / 240$
- C) $240 / 240$
- D) $120 / 240$
- E) $90 / 120$

22 **Error Analysis.** Asked “Among the students who prefer texting, what proportion are underclassmen?” a student writes “ $90/120 = 0.75$.” Explain the mistake (which group should the denominator be?) and give the correct value.

★★★

USE THIS TABLE FOR PROBLEM 23

A school recorded whether students joined at least one club this year, by grade. Each grade has 80 students.

	9TH	10TH	11TH	TOTAL
Joined a club	48	40	24	112
Did not	32	40	56	128
TOTAL	80	80	80	240

23 Find the conditional relative frequency of joining a club for *each* grade. Then describe how club membership changes across grades and whether grade and joining a club appear associated.

★★★

- 24** ★★★★★ Using the texting/calling table (problems 17–22), a student council claims: “Underclassmen prefer texting more than upperclassmen do.”
- a) Compute the conditional distribution of preference for each class level.
 - b) Use your values to decide whether the data support the council’s claim.
 - c) Explain why this table cannot show that *being* an underclassman *causes* a texting preference.

- 25** ★★★★★ Using the texting/calling table, the marginal relative frequency of preferring *calling* is

★★★★★



- A) 90 / 240
- B) 60 / 120
- C) 30 / 120
- D) 60 / 90
- E) 90 / 150

- 26** ★★★★★ School A has 400 students and School B has 100 students. A bar chart of *raw counts* shows that more students at School A bike to school, and a reporter concludes “School A students like biking far more.” Explain why comparing raw counts is misleading here, and state what should be compared instead.

- 27** ★★★★★ **True or False.** For any you mark false, give a one-line reason.

★★★★★

- a) In a mosaic plot, the widths of the columns show the marginal distribution of one variable.
- b) If two categorical variables are associated, the conditional distributions of one variable are identical across the categories of the other.
- c) A segmented bar chart scaled to 100% displays conditional distributions.

28

★★★

Matching. Using the texting/calling table, match each value to what it represents. Each is used once.

- | | |
|----------------|---|
| 1. $90 / 240$ | A. Conditional: among underclassmen, the proportion who prefer texting |
| 2. $150 / 240$ | B. Joint: the proportion who prefer texting and are underclassmen |
| 3. $90 / 120$ | C. Marginal: the proportion who prefer texting |
| 4. $90 / 150$ | D. Conditional: among texting-preferers, the proportion who are underclassmen |
-
-

Chance & Simulation

Random processes, outcomes & events · probability as a long-run relative frequency · the Law of Large Numbers · designing & running simulations

29 **True or False.** For any you mark false, give a one-line reason.

★★★★

- a) A random process generates results determined by chance.
- b) An *outcome* is a collection of one or more events.
- c) The probability of an event is its long-run relative frequency over many trials.

30 The probability that a spun bottle points at a particular tile is best described as

★★★★



- A) the result of a single spin.
- B) the proportion of spins pointing at that tile over a very large number of spins.
- C) a number that can be larger than 1.
- D) guaranteed to occur exactly once every ten spins.
- E) impossible to estimate from data.

31 A basketball player makes 60% of her free throws. You want to simulate her next free throw using a single random digit (0–9).

★★★★

- a) Describe how to assign the digits 0–9 to the outcomes “make” and “miss” so the simulation matches her ability.
- b) Explain why this digit assignment correctly models a 60% chance of making the shot.

32 Mia repeatedly spins a prize wheel that lands on “win” 40% of the time. After each spin she plots the cumulative proportion of wins so far. After 12 spins the proportion is 0.25; after 400 spins it is 0.41.

★★★★

- a) Explain what the **Law of Large Numbers** says will happen to the cumulative proportion of wins as the number of spins keeps increasing.
- b) Does the Law of Large Numbers guarantee that the *number* of wins will be exactly 40% of the spins at some point? Explain.

33

★★★

MC

Which statement best describes the **Law of Large Numbers**?

- A) After a long run of heads, tails becomes more likely to balance things out.
- B) Large samples always give a proportion exactly equal to the true probability.
- C) As the number of independent trials increases, the long-run relative frequency of an outcome approaches a single value.
- D) Probabilities change as more data are collected.
- E) Every outcome in a sample space is equally likely.

34

★★★

A claw machine releases a prize 15% of the time. You will simulate playing until the first prize using pairs of random digits, where 01–15 = “prize” and 16–00 (i.e., 16–99 and 00) = “no prize.” One trial = read pairs until a prize, then record how many plays it took.

- a) Using the random digits below, carry out **two** trials and record the number of plays each took.
- b) Based only on these two trials, estimate the average number of plays to win a prize.

87 42 09 56 31 15 68 73 04 90 22 11 47 80 63

35

★★★

A class wanted to estimate the probability that, when 5 friends each randomly pick one of several group playlists, *at least two* friends pick the same playlist. They ran 40 simulated groups and found that 26 of the groups had at least two friends choose the same playlist.

- a) Use the simulation to estimate this probability.
- b) Explain why this estimate would likely change if the class ran only 8 groups instead of 40, and which run you would trust more.

36

★★★

Error Analysis. A gamer says, “This loot box has a 10% chance of a rare item. I’ve opened 9 in a row with nothing, so the next one is basically guaranteed to be rare.” Identify the misconception and explain what the probability of a rare item is on his next box, and why.

- 37** **True or False.** For any you mark false, give a one-line reason.
★★★☆☆
- A simulation's estimated probability is guaranteed to exactly equal the true probability.
 - Running more trials makes a simulation's estimate tend to land closer to the true probability.
 - A probability can be negative.
-
-
-
- 38** A new phone game advertises that 30% of mystery chests contain gems. A player wants to estimate the probability that, in 5 chests, **at least 3** contain gems.
★★★★★
- Describe a simulation: how would you use single random digits to model one chest, and what makes up one complete trial?
 - State exactly what you would record after each trial in order to estimate the probability.
-
-
- 39** A relative frequency from collected data is used to **estimate** a true probability. As the amount of data increases, that relative frequency
★★★★★
- MC**
- moves further from the true probability.
 - stops changing after the first few trials.
 - becomes exactly 0.5.
 - tends to settle near the true probability.
 - can exceed 1 if the event is common.
-
- 40** A streaming app finds that the probability a randomly chosen new viewer finishes a show within a week is 0.20. The best interpretation of 0.20 is:
★★★★★
- MC**
- Exactly 20% of any 5 viewers will finish it within a week.
 - Each viewer is required to finish 20% of the show.
 - The next viewer will finish it in 0.2 weeks.
 - Exactly 20 of the next 100 viewers will finish it.
 - Over very many viewers, about 20% finish it within a week.
-
- 41** A claw machine releases a prize on 35% of plays. To simulate one play using a two-digit random number (00–99), which assignment works?
★★★★★
- MC**
- 01–35 = prize; 36–99 and 00 = no prize.
 - Even numbers = prize; odd numbers = no prize.
 - 0–3 = prize; 4–9 = no prize.
 - 35–99 = prize; 00–34 = no prize.
 - 01–65 = prize; 66–99 and 00 = no prize.

USE THIS SIMULATION FOR PROBLEM 42

Each of 4 friends independently has a 25% chance of winning a prize in a game. A class ran 50 trials of the game and recorded the number of friends who won in each trial.

	NUMBER WHO WON	0	1	2	3	4
Number of trials		16	21	9	3	1

- 42** Using the simulation results, estimate the probability that **at least 2** of the 4 friends win. Show the counts you used.

★★★★

- 43** A fair coin has just landed heads 6 times in a row. The probability that the next flip is heads is

★★★

MC

- A) less than 0.5, because tails is “due.”
- B) 0.5, because the flips are independent.
- C) greater than 0.5, because heads is on a streak.
- D) 0, because it can’t happen 7 times.
- E) 1, because heads always wins.

- 44** A student rolls a fair die 10,000 times and tracks the proportion of rolls that come up 6. The proportion getting closer and closer to $1/6$ illustrates the

★★★

MC

- A) addition rule.
- B) complement rule.
- C) law of large numbers.
- D) multiplication rule.
- E) property of mutually exclusive events.

- 45** A cereal brand puts a prize in 20% of boxes. You want to estimate the probability that a family buying 4 boxes gets *at least one* prize.

★★★★

- a) Assign random digits (0–9) to model one box.
- b) Describe what makes up one complete trial.
- c) State exactly what you would record after each trial to make the estimate.

46

★★★

True or False. For any you mark false, give a one-line reason.

- a) A probability of 0.5 means the event happens on every other trial.
 - b) A simulation that uses only 5 trials gives a reliable estimate of the true probability.
 - c) A simulation can estimate a probability that would be hard to calculate exactly.
-
-
-

47

★★★

One class's 30-trial simulation estimated a probability as $7/30 \approx 0.23$; another class's 30-trial simulation got 0.30. Explain why two correct simulations can give different estimates, and what would make the two estimates more similar.

Basic Probability Rules

Sample spaces & equally likely outcomes · theoretical probability · the complement rule · mutually exclusive (disjoint) events & joint probability

- 48** ★★★ A spinner has four equal regions colored red, blue, green, and yellow. (a) List the sample space for one spin. (b) What is the probability the spinner lands on green? (c) What is the probability it does *not* land on green?

- 49** ★★★ If $P(\text{a student picks chocolate}) = 0.42$, then the probability the student does *not* pick chocolate is



- A) 0.42
- B) 1.42
- C) 0.84
- D) 0.58
- E) 0

- 50** ★★★ Two fair six-sided dice are rolled and the numbers are added.

- a) How many equally likely outcomes are in the sample space?
- b) Find the probability the sum is 7.
- c) Find the probability the sum is *not* 7, using the complement rule.

- 51** ★★★ For one randomly chosen song on a playlist, $P(\text{pop}) = 0.35$, $P(\text{hip-hop}) = 0.30$, and $P(\text{indie}) = 0.20$; every song has exactly one genre. (a) Find the probability the song is *none* of these three genres. (b) Find the probability the song is pop or hip-hop.

52

★★★★★



Two events A and B are **mutually exclusive** (disjoint). This means that

- A) $P(A) = P(B)$.
- B) A and B are the same event.
- C) knowing A occurs does not change the probability of B.
- D) $P(A) + P(B) = 1$ always.
- E) they cannot occur at the same time, so $P(A \cap B) = 0$.

53

★★★★★

When a single card is drawn from a standard 52-card deck, let A = “the card is a king” and B = “the card is a queen.” (a) Find $P(A \cap B)$. (b) Are A and B mutually exclusive? Justify your answer using your value from part (a).

54

★★★★★

Error Analysis. A student writes: “Getting an A on the quiz and getting a B on the quiz are mutually exclusive, so they must also be independent.” Explain why “mutually exclusive” and “independent” are not the same idea, and why two mutually exclusive events (each with probability > 0) cannot be independent.

55

★★★★★

A bag holds 5 mint, 3 sour, and 2 chili gummies (10 total). One gummy is drawn at random. (a) Find $P(\text{mint})$. (b) Find $P(\text{not chili})$. (c) Are “mint” and “sour” mutually exclusive? Explain.

56

★★★★★



Which of the following could **not** be a probability?

- A) 1.3
- B) 0.07
- C) 1
- D) 0
- E) 0.95

57 **True or False.** For any you mark false, give a one-line reason.

★★★

- a) The probabilities of all outcomes in a sample space add to 1.
- b) If two events are mutually exclusive, $P(A \cap B) = 0$.
- c) Two events that can both happen at once are mutually exclusive.
- d) The complement of an event has probability 1 minus the event's probability.

58 In a club, $P(\text{plays a sport}) = 0.6$ and $P(\text{plays an instrument}) = 0.5$. A student claims these two events must be mutually exclusive. Explain why that claim cannot be correct, using the probabilities given.

★★★

59 A six-sided die is rolled once. Let $A = \text{"roll an even number"}$ and $B = \text{"roll a 5."}$ Which statement is true?

★★★



- A) A and B both occur when you roll a 4.
- B) A and B are mutually exclusive, because no roll is both even and a 5.
- C) $P(A) = 1/6$.
- D) $P(B) = 1/2$.
- E) A is the complement of B.

60 A school raffle sells tickets in four colors. For a randomly drawn winning ticket, $P(\text{blue}) = 0.30$, $P(\text{green}) = 0.25$, $P(\text{orange}) = 0.20$, and the rest are pink. Each ticket is exactly one color.

★★★

- a) Find $P(\text{pink})$. Show how the complement / total-probability idea gives your answer.
- b) Find the probability the winning ticket is blue or orange. Explain why you may add the two probabilities here.
- c) Are "blue" and "green" mutually exclusive? Are they independent? Briefly justify each.

61 **Matching.** Match each term to its description. Each is used once.

★★★

- | | |
|-----------------------|--|
| 1. Sample space | A. $P(A \cap B)$, the chance both events happen |
| 2. Complement of E | B. the set of all possible nonoverlapping outcomes |
| 3. Mutually exclusive | C. two events that cannot happen together |
| 4. Joint probability | D. "not E," with probability $1 - P(E)$ |

- 62 Which of the following **cannot** be a probability?
★★★★★
MC
- A) 0
 - B) 0.3
 - C) 1.4
 - D) 1
 - E) 0.95
-
- 63 In a phone-case shop, each case is exactly one color. $P(\text{black}) = 0.30$, $P(\text{clear}) = 0.20$, and $P(\text{pink}) = 0.15$. Which of the following **cannot** be the probability that a randomly chosen case is teal?
★★★★★
MC
- A) 0
 - B) 0.10
 - C) 0.20
 - D) 0.35
 - E) 0.45
-
- 64 Each loot box has a 0.10 chance of containing a rare item, independently. Find the probability of getting **at least one** rare item in 3 boxes. Write the calculation using the complement.
★★★★★
-
-
- 65 You flip a fair coin and roll a fair four-sided die (faces 1–4). (a) How many equally likely outcomes are in the sample space? (b) Find $P(\text{heads and a 4})$. (c) Find $P(\text{the die shows an even number})$.
★★★★★
-
-
- 66 A game designer says the probability that a random player reaches level 10 is 0.40. The best interpretation is:
★★★★★
MC
- A) Exactly 4 of the next 10 players will reach level 10.
 - B) The next player has a 40-minute wait.
 - C) Every player has reached level 10 forty percent of the way.
 - D) Over very many players, about 40% reach level 10.
 - E) Reaching level 10 is impossible for 60% of players.

67

★★★★

A group chat has six members. The app will randomly choose **two** of them to be admins.

- a) How many different pairs of two members are possible?
- b) Find the probability that Maya and Theo are the pair chosen.

68

★★★★



One card is drawn from a standard deck. Which pair of events is **mutually exclusive**?

- A) "The card is a heart" and "the card is a spade."
- B) "The card is red" and "the card is a king."
- C) "The card is a face card" and "the card is a queen."
- D) "The card is a club" and "the card is a 7."
- E) "The card is black" and "the card is an ace."

69

★★★★



For two events A and B that each have a probability greater than 0, which statement is **always** true?

- A) If A and B are independent, then they are mutually exclusive.
- B) If A and B are mutually exclusive, then they cannot be independent.
- C) If A and B are mutually exclusive, then $P(A \cap B) = P(A) \cdot P(B)$.
- D) Two mutually exclusive events always satisfy $P(A) + P(B) = 1$.
- E) If A and B are mutually exclusive, then $P(A \cup B) = P(A) \cdot P(B)$.

70

★★★★

A new driver passes the road test on the first try with probability 0.70. (a) Find the probability of *not* passing on the first try. (b) For two independent new drivers, find the probability that *neither* passes on the first try.

71

★★★★



For two events with $P(A) = 0.5$ and $P(B) = 0.4$, which value of $P(A \cap B)$ would make A and B mutually exclusive?

- A) 0.20
- B) 0.45
- C) 0
- D) 0.50
- E) 0.90

- 72** ★★★ A spinner is divided into four regions A, B, C, and D. $P(A)$ is twice $P(B)$, $P(C) = 0.25$, and $P(D) = 0.15$. The four probabilities must add to 1. Find $P(A)$ and $P(B)$. Show your reasoning.

- 73** ★★★ At a fair, each guest selects exactly one prize: a plush, a poster, or a pin. Let A = “a guest picks the plush” and B = “a guest picks the poster.” Are A and B mutually exclusive? Justify your answer using $P(A \cap B)$.

- 74** ★★★ A basketball player makes each free throw with probability 0.80, independently. **(a)** Find the probability she makes *at least one* of her next 3 free throws. **(b)** Explain why using the complement is easier than adding up cases here.

- 75** ★★★ If the probability that a randomly selected song on a playlist is explicit is 0.18, then the probability that the song is *not* explicit is



- A) 0.18
- B) 1.18
- C) 0.50
- D) 0.82
- E) 0

Conditional Probability & Independence

Conditional probability from tables & the formula · the general multiplication rule · testing whether two events are independent

76 **True or False.** For any you mark false, give a one-line reason.

★★★★

- a) $P(A \mid B)$ is read “the probability of A given that B has occurred.”
- b) $P(A \mid B) = P(A \cap B) / P(B)$.
- c) $P(A \mid B)$ and $P(B \mid A)$ always have the same value.

USE THIS TABLE FOR PROBLEMS 77–80

A tech blog surveyed 200 phone owners, recording phone type and whether the owner keeps the phone in dark mode.

	IPHONE	ANDROID	TOTAL
Dark mode: Yes	48	52	100
Dark mode: No	72	28	100
TOTAL	120	80	200

77 Find each conditional probability. (a) $P(\text{dark mode} \mid \text{iPhone})$. (b) $P(\text{dark mode} \mid \text{Android})$. Show the calculation for each.

★★★★

78 Find $P(\text{iPhone} \mid \text{dark mode})$. Explain in one sentence why this is a different question from $P(\text{dark mode} \mid \text{iPhone})$.

★★★★

79 Are “dark mode” and “owning an iPhone” **independent** events for these owners? Decide by comparing $P(\text{dark mode} \mid \text{iPhone})$ with $P(\text{dark mode})$, and explain what your comparison shows.

★★★★

80 Using the table, $P(\text{Android} \mid \text{dark mode})$ is

★★★★



- A) $52 / 200$
- B) $80 / 200$
- C) $52 / 80$
- D) $100 / 200$
- E) $52 / 100$

81 A snack bin holds 6 granola bars and 4 fruit cups (10 total). A student grabs two snacks one after the other, *without replacement*. Using the general multiplication rule, find (a) $P(\text{both are granola bars})$ and (b) $P(\text{the first is a granola bar, then a fruit cup})$. Write each as an expression before computing.

★★★★

82 Events A and B are **independent** if and only if

★★★★



- A) $P(A \mid B) = P(A)$.
- B) $P(A) = P(B)$.
- C) $P(A) + P(B) = 1$.
- D) A and B cannot occur together.
- E) $P(A \cap B) = 0$.

83 **Error Analysis.** A weather app states $P(\text{rain} \mid \text{dark clouds}) = 0.80$. A student concludes that $P(\text{dark clouds} \mid \text{rain})$ is also 0.80. Explain why these two conditional probabilities need not be equal, using the idea of which group you are restricting to in each case.

★★★★

84 At a gym, $P(\text{takes a spin class}) = 0.4$, $P(\text{takes yoga}) = 0.5$, and $P(\text{takes spin} \cap \text{yoga}) = 0.2$.

★★★★

- a) Find $P(\text{yoga} \mid \text{spin})$.
- b) Are “spin” and “yoga” independent? Justify by comparing $P(\text{yoga} \mid \text{spin})$ with $P(\text{yoga})$.

85

Always / Sometimes / Never. Write **A**, **S**, or **N** with a one-line reason.

★★★

- a) If A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$.
 b) $P(A | B)$ equals $P(B | A)$.
 c) If $P(A | B) = P(A)$, then A and B are independent.

86

For two events with $P(A) = 0.5$ and $P(B) = 0.4$, which condition would show A and B are independent?

★★★



- A) $P(A \cap B) = 0$
 B) $P(A \cap B) = 0.20$
 C) $P(A \cap B) = 0.9$
 D) $P(A \cup B) = 0.5$
 E) $P(A | B) = 0.4$

87

A coffee chain finds that 70% of morning customers order a drink, 40% order food, and 30% order both.

★★★★

- a) Given a customer orders food, what is the probability they also order a drink?
 b) Are ordering a drink and ordering food independent? Justify with a calculation.
 c) Interpret your answer to (b) in plain language about morning customers.

VENN DIAGRAM — DRINK & FOOD

88

A phone's Face ID unlocks for the correct user 80% of the time on each independent attempt. (a) Find the probability it unlocks on both of the next two attempts. (b) Find the probability it fails on both of the next two attempts. Write each as an expression first.

★★★★

USE THIS TABLE FOR PROBLEMS 89–92

A survey of 300 students recorded each student's main weekend activity and whether they got 8 or more hours of sleep on a typical weekend night.

	GAMING	SPORTS	WORK	TOTAL
8+ hours of sleep	60	50	40	150
Fewer than 8	40	30	80	150
TOTAL	100	80	120	300

- 89** Find each conditional probability. (a) $P(8+ \text{ hours of sleep} \mid \text{works on weekends})$. (b) $P(\text{works} \mid \text{got fewer than 8 hours})$. Show each calculation.

★★★

- 90** Is “getting 8+ hours of sleep” **independent** of “working on weekends”? Decide by comparing $P(8+ \mid \text{works})$ with $P(8+)$, and explain.

★★★

- 91** Given that a student did *not* choose gaming as their main activity, what is the probability they got 8+ hours of sleep? Show the counts you used.

★★★

- 92** Using the table, $P(\text{gaming} \mid 8+ \text{ hours of sleep})$ is

★★★

- A) 60 / 300
- B) 60 / 100
- C) 100 / 300
- D) 60 / 150
- E) 150 / 300

MC

93 ★★★ A screening test for a banned substance is given to 100 athletes, 10 of whom actually used it. Of users, 90% test positive. Of non-users, 20% test positive (a false positive).

- a) Complete the two-way table of counts (used / did not, by test result).
 b) Find the probability that an athlete who tests positive actually used the substance, $P(\text{used} \mid \text{positive})$.

	POSITIVE	NEGATIVE	TOTAL
Used			
Did not use			
TOTAL			

94 ★★★★★ An email filter knows that 40% of incoming messages are spam. It flags 95% of spam messages and also flags 8% of real (non-spam) messages.

- a) Draw a tree diagram and find the probability that a randomly chosen message is flagged.
 b) Find the probability that a flagged message is actually spam, $P(\text{spam} \mid \text{flagged})$.

95 ★★★★★ A bag holds 3 cherry and 2 lime candies. Two candies are drawn at random *with replacement*. (a) Find $P(\text{both cherry})$. (b) Find $P(\text{first cherry, then lime})$. Write each as an expression first.

96 ★★★★★ Using the same bag (3 cherry, 2 lime), now draw two candies *without replacement*. (a) Find $P(\text{both cherry})$. (b) Explain why this is smaller than the with-replacement answer from problem 95.

97

★★★

Suppose $P(A) = 0.6$ and $P(B) = 0.5$. Events A and B are independent exactly when $P(A \cap B)$ equals



- A) 0
- B) 0.11
- C) 0.30
- D) 0.55
- E) 1.10

98

★★★

A fair die with faces 1–6 is rolled once. (a) Given that the result is even, find the probability it is a 2. (b) Given that the result is greater than 3, find the probability it is a 6.

99

★★★

A study reports that 35% of students who pulled an all-nighter before a test failed it. Letting F = “failed” and N = “pulled an all-nighter,” this 35% is best written as



- A) $P(N | F) = 0.35$
- B) $P(N) = 0.35$
- C) $P(F \cap N) = 0.35$
- D) $P(F) = 0.35$
- E) $P(F | N) = 0.35$

100

★★★

A student independently makes the soccer team with probability 0.5 and the debate team with probability 0.4. Find the probability that the student makes *exactly one* of the two teams. Show your work.

101

★★★

For two events, $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.15$. Determine whether A and B are independent by performing the appropriate check.

102

★★★

Explain which is likely larger: $P(\text{speaks Spanish} | \text{in the Spanish club})$ or $P(\text{in the Spanish club} | \text{speaks Spanish})$. Justify your answer by describing the group each probability restricts to.

103

★★★



Two cards are drawn from a standard 52-card deck without replacement. The probability that both are aces is

- A) $(4/52)(3/51)$
- B) $(4/52)(4/52)$
- C) $(4/52)(3/52)$
- D) $(4/52) + (3/51)$
- E) $(2/52)(1/51)$

104

★★★★

At a school, 30% of students take art, 25% take music, and 10% take both.

- a) Find $P(\text{takes music} \mid \text{takes art})$.
- b) Are taking art and taking music independent? Justify with a calculation.
- c) Find the probability a student takes *exactly one* of the two subjects.

105

★★★



To check whether events A and B are independent, the value of $P(A \mid B)$ should be compared with

- A) $P(B)$.
- B) $P(A)$.
- C) $P(A \cup B)$.
- D) $P(B \mid A)$ only.
- E) $1 - P(B)$.

Unions, the General Addition Rule & Tree Diagrams

The general addition rule · why we subtract the intersection · Venn diagrams · tree diagrams & multi-step probability

106

★★★★



The **general addition rule** for the probability that A or B (or both) occurs is

A) $P(A \cup B) = P(A) \cdot P(B)$.

B) $P(A \cup B) = P(A) + P(B)$.

C) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

D) $P(A \cup B) = P(A) + P(B) + P(A \cap B)$.

E) $P(A \cup B) = 1 - P(A) - P(B)$.

107

★★★★

For one student, $P(\text{takes Spanish}) = 0.45$, $P(\text{takes art}) = 0.30$, and $P(\text{takes both}) = 0.15$. Find the probability the student takes Spanish or art (or both). Write the rule with numbers substituted in.

108

★★★★

When a single card is drawn from a standard deck, find $P(\text{king or queen})$. Explain why, in this case, the general addition rule reduces to simply adding the two probabilities.

109

★★★★

Error Analysis. To find $P(\text{senior or plays a sport})$ at a school, a student adds $P(\text{senior}) = 0.25$ and $P(\text{plays a sport}) = 0.40$ to get 0.65, but the true value is 0.55. Explain what the student forgot to do, and find $P(\text{senior} \cap \text{plays a sport})$.

110

★★★★

For two events, $P(A) = 0.50$, $P(B) = 0.40$, and $P(A \cap B) = 0.20$. Fill in a Venn diagram and use it to find: (a) $P(A \text{ only})$, (b) $P(B \text{ only})$, (c) $P(\text{neither } A \text{ nor } B)$.

111

★★★★



For events with $P(A) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$, the probability of $A \cup B$ is

- A) 0.9
- B) 0.12
- C) 0.5
- D) 0.7
- E) 1.1

112

★★★★

Always / Sometimes / Never. Write **A**, **S**, or **N** with a one-line reason.

- a) $P(A \cup B) = P(A) + P(B)$.
- b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- c) For mutually exclusive events, $P(A \cup B) = P(A) + P(B)$.

113

★★★★

On a school morning, the probability a student wakes up late is 0.30. If late, the probability they miss the bus is 0.50; if not late, the probability they miss the bus is 0.10.

- a) Draw a tree diagram for this situation, labeling each branch with its probability.
- b) Find the probability the student misses the bus.

114

★★★★

A drawer has 3 black socks and 2 white socks. Two socks are drawn at random *without replacement*.

- a) Draw a tree diagram showing the two draws, with the correct probability on every branch.
- b) Find the probability the two socks **match** (both black or both white).

115

★★★

A two-way table gives $P(\text{plays guitar}) = 0.30$, $P(\text{plays piano}) = 0.25$, and $P(\text{plays both}) = 0.10$. The probability a student plays guitar or piano is

MC

- A) 0.55
- B) 0.10
- C) 0.65
- D) 0.075
- E) 0.45

116

★★★★

At a bubble-tea shop, 70% of orders are cold drinks and 30% are hot. Of cold drinks, 40% add boba; of hot drinks, 20% add boba. One order is chosen at random.

- a) Draw a tree diagram and find the probability the order has boba.
- b) Given that an order has boba, find the probability it was a cold drink.
- c) Explain why your answer to (b) is different from the 40% “cold $\rightarrow$ boba” rate.

117

★★★★

Capstone. A music app classifies each new user as Free (75%) or Premium (25%). In their first week, 20% of Free users and 60% of Premium users create a playlist.

- a) Find the probability a randomly chosen new user creates a playlist in week 1.
- b) Find the probability a new user is Premium *and* creates a playlist.
- c) Given that a user created a playlist, find the probability they are a Premium user.
- d) Are “Premium” and “created a playlist” independent? Justify with the numbers.

A tree diagram may help organize the two stages.

118

★★★

At a mall, for a random teen shopper $P(\text{visits the food court}) = 0.60$, $P(\text{visits the arcade}) = 0.35$, and $P(\text{visits both}) = 0.20$. Find the probability the shopper visits the food court or the arcade (or both). Write the rule with numbers substituted in.

119

★★★

A club has 30 members. Of them, 12 like pizza, 9 like tacos, and 5 like both. One member is chosen at random. Find the probability the member likes pizza or tacos.

120

★★★



Events M and N are mutually exclusive with $P(M) = 0.30$ and $P(N) = 0.45$. The probability of M or N is

- A) 0.135
- B) 0.15
- C) 0.75
- D) 0.90
- E) 1.05

121

★★★

For two events, $P(A) = 0.45$, $P(B) = 0.30$, and $P(A \cap B) = 0.15$. Fill in a Venn diagram and use it to find: (a) $P(A \text{ only})$, (b) $P(\text{neither A nor B})$, (c) $P(B | A)$.

122

★★★

Two friends text a group chat independently each morning: friend A with probability 0.7 and friend B with probability 0.6. Find the probability that *at least one* of them texts. Show the calculation (complement or addition rule).

123

★★★★

For two events, $P(A) = 0.50$, $P(B) = 0.40$, and $P(A \cap B) = 0.25$. Find the probability that *exactly one* of A or B occurs. Show your work.

124

★★★★

When one card is drawn from a standard deck, find $P(\text{heart or club})$. Explain why the general addition rule reduces to simply adding the two probabilities here.

125

★★★★

A factory uses two machines. Machine A makes 60% of the parts and Machine B makes 40%. Machine A's defect rate is 2%; Machine B's is 5%.

- a) Draw a tree diagram and find the probability a randomly chosen part is defective.
- b) Given that a part is defective, find the probability it was made by Machine B.

126

★★★★

A bag holds 2 gold and 3 silver coins. Two coins are drawn at random *without replacement*.

- a) Draw a tree diagram with the correct probability on every branch.
- b) Find the probability of getting *at least one* gold coin.

127

★★★

Error Analysis. To find $P(\text{senior or athlete})$, a student adds $P(\text{senior}) = 0.30$ and $P(\text{athlete}) = 0.45$ to get 0.75, but the true value is 0.60. Explain what the student forgot.

128

★★★★

At a school, 60% of students have a part-time job, 50% have a car, and 35% have both.

- a) Find $P(\text{has a job or a car})$.
- b) Find $P(\text{has a car} \mid \text{has a job})$.
- c) Are having a job and having a car independent? Justify.
- d) Find $P(\text{has neither})$.

129

★★★★



For two events with $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$, the probability that *neither* A nor B occurs is

- A) 0.1
- B) 0.3
- C) 0.5
- D) 0.7
- E) 0.9

130

★★★★

In the factory of problem 125, the probability a defective part came from Machine B works out to be larger than Machine B's own 5% defect rate. Explain why these two numbers answer different questions.

131

★★★★

A student applies to three colleges. The admission decisions are independent, with probabilities 0.5, 0.4, and 0.3. Find the probability the student is admitted to *at least one* college. Use the complement.

132

★★★★

Capstone. At a school, 75% of students follow the school's social account, 40% entered a giveaway, and 35% did both.

- a) Find $P(\text{follows the account or entered the giveaway})$.
- b) Find $P(\text{entered the giveaway} \mid \text{follows the account})$.
- c) Are following the account and entering the giveaway independent? Justify.
- d) Find the probability a student did *exactly one* of the two.
- e) Find the probability a student did *neither*.